

## PLENARIES

### TUESDAY

#### TITLE

Cable-Operated Elevators and Deep-Sea Construction: 4x4 Hyperbolic PDE-ODE Control with Moving Boundary

#### AUTHORS

Miroslav Krstic

#### ABSTRACT

### WEDNESDAY

#### TITLE

Boundary stabilization of 1-D hyperbolic systems

#### AUTHOR

Jean-Michel Coron

#### ABSTRACT

Hyperbolic systems in one space dimension appear in various real life applications (navigable rivers and irrigation channels, heat exchangers, plug flow chemical reactors, gas pipe lines, chromatography, traffic flow,...). This talk will focus on the stabilization of these systems by means of boundary controls. Stabilizing feedback laws will be constructed. This includes explicit feedback laws which have been implemented for the regulation of the rivers La Sambre and La Meuse. The talk will also deal a little bit with the more complicate case where there are source terms.

### THURSDAY

#### TITLE

Heat equations with memory: flow decomposition, constraints and control

#### AUTHOR

Enrique Zuazua

- Friedrich-Alexander-Universität Erlangen-Nürnberg/Alexander von Humboldt Professorship, Germany
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- Joint work with: Felipe Chaves-Silva, Qi Lü, Lionel Rosier, Gengsheng Wang, Xu Zhang, and Yubiao Zhang

## ABSTRACT

We analyze the controllability properties of heat equations involving memory terms. We interpret the system as, roughly, the coupling of a heat equation with an ordinary differential equation (ODE). The presence of the ODE for which there is no propagation along the space variable makes the controllability of the system impossible when the control is confined into a subset in space that does not move. Following [1], [2] and [4], the null controllability of the system with a moving control is established using the observability of the adjoint system and some Carleman estimates for a coupled system of a parabolic equation and an ODE with the same singular weight, adapted to the geometry of the moving support of the control. This extends to the multi-dimensional case the results of [5] on the one-dimensional case, employing 1-D Fourier analysis techniques. We also show how the techniques in [3] and [6] can be adapted to handle the same problems under pointwise constraints on states and controls. In the case of time-dependent analytic kernels, following [7], we also describe how the flow can be decomposed into a parabolic and a hyperbolic component, which allows to analyze from close the regularizing effect of the evolution and the propagation of singularities.

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## SESSIONS

## Tuesday Morning

### TITLE

Delay Robustness Optimization on First-Order Consensus under PID Protocols: Directed Graphs

### AUTHORS

Dan Ma (corresponding author)

Jie Chen

### ABSTRACT

Multi-Agent Systems (MASs) seek to connect individual agents or subsystems via a communication network so as to perform a joint task. Consensus is a central problem for multi-agent systems, which amounts to designing a suitable feedback control protocol so that all the agents in the network converge to a common state. Much as time delay has negative effect on a system's stability and performance in general, it has been well-known that the agent and inter-agent communication delays impede consensus attainment. Since by nature delays in MASs are typically uncertain and difficult to estimate, it is necessary to address consensus robustness under the effect of uncertain, possible variable delays. Toward this end, the existing results consist of by-and-large bounds on the delay range, and are largely limited to undirected network graphs. Determination of the exact delay robustness range, while much desired and at times necessary, proved far more elusive and has been largely an open problem.

In this chapter we study delay robustness of multi-agent systems (MASs) under PID protocols, focusing on more complex networks described by directed graphs. PID control is time-honored and remains to be the favored control law in conventional feedback design and implementation. For its well-recognized advantages, it is both natural and tempting to examine the utility of PID protocols in the MAS setting. We consider first-order agents and dynamic feedback control protocols in the form of PID control, subject to an uncertain constant delay, for which we seek to determine the delay consensus margin (DCM), i.e., the exact delay robustness range. The DCM, which generally poses a non-smooth max-min problem, is shown to be computable by solving a convex optimization problem. Moreover, we also prove that the DCM achievable by PID protocols coincides with that by PD protocols. Our analytical results demonstrate how unstable agent dynamics and graph connectivity may fundamentally limit the tolerable range of delay, so that consensus can or cannot be maintained in the presence of delay variations.

### REFERENCES

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## TITLE

Cable-Operated Elevators and Deep-Sea Construction: 4x4 Hyperbolic PDE-ODE Control with Moving Boundary

## AUTHORS

Miroslav Krstic

Ji Wang

## ABSTRACT

In mineral exploitation (mining), a cable-operated elevator is used to transport the cargo and miners between the ground and the working platform underground. A dual-cable mining elevator is used in the operation at a great depth, such as over 2000 m, and carrying a heavy load, where the single-cable elevator is not suitable, because a very thick cable, which is required to bear the heavy load, suffers from problems in the winding on the winder drum, due to its high bending. A deep-sea construction vessel, is used to place underwater parts of an off-shore oil drilling platform at the designated locations on the seabed, and is also used in the construction of artificial reefs for enabling the development and growth of marine life, in laying communication cables on the seabed, which is often uneven and rife with obstacles, and in other undersea applications. The dynamics of the mining cable elevator and deep-sea construction vessel are described by 4x4 coupled hyperbolic PDE systems, which are converted from two wave PDEs by the Riemann transformation, on a time-varying spatial domain introduced by the time-varying length of the cable, and coupled with an ODE which describes the dynamics of the attached payload, i.e., the cage or object to be installed/recycled/moved on the seabed. In the dual-cable mining elevator, we deal with axial oscillations in a pair of

cables connected by a payload at the distal boundary. So the 4x4 system in the cable elevator is a set of two 2x2 pairs that are not coupled along the domain but are coupled at the boundary. In contrast, in the deep-sea construction vessel, we deal with a single cable but with axial and lateral oscillations which bring domain-wide coupling into play and, therefore, a fully coupled 4x4 hyperbolic system. The boundary control inputs, i.e., control forces applied at the head sheave of the mining cable elevator and the ship-mounted crane of the deep-sea construction vessel, are designed by using the backstepping method. The simulations verify that the designed controllers achieve suppression of axial vibrations and tension oscillations in the cables, as well as balancing the cage roll, in the dual-cable mining elevator, and also effectively reduce the undesired longitudinal-lateral vibrations in the deep-sea construction vessel with ocean current disturbances, even though the design is developed in the absence of such real-world effects.

## TITLE

Hybrid boundary stabilization of linear first-order hyperbolic PDEs despite almost quantized measurements and control input

## AUTHOR

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## ABSTRACT

Despite the results [1], [5], in which practical stabilization is achieved (as the considered quantizers are static and the control design approaches do not, explicitly, aim at treating state measurements errors due to quantization), the problem of compensation of quantization in measurements and control input for boundary controlled, linear, first-order hyperbolic PDE systems, achieving global asymptotic stabilization, has not been, heretofore, investigated. We construct a hybrid feedback law, which is based on combination of two elements—A nominal backstepping control design [3], which achieves stabilization of the PDE system in the absence of quantization, and a switching update law for the tunable parameter of the quantizer [4], which achieves compensation of the quantization effect. Global asymptotic stability of the closed-loop system is established through utilization of Lyapunov-like arguments and derivation of solutions' estimates, providing explicit estimates for the supremum norm of the PDE state, capitalizing on the relation of the resulting, nonlinear PDE system (in closed loop) to a certain, integral delay equation [2]. Although the central design and analysis ideas are similar to the case of state quantization, we also develop a respective hybrid feedback law for the case in which state measurements are available, yet, the control input signal is subject to quantization. A numerical example is also provided to illustrate, in simulation, the effectiveness of the developed design. Furthermore, we discuss possible research problems that could, potentially, draw inspiration from the presented results.

## REFERENCES

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## Tuesday Afternoon

### TITLE

Numerical control of the semilinear wave equation on a bounded interval.

### AUTHORS

Lionel Rosier,

M.M. Cavalcanti,

V.N. Domingos-Cavalcanti, and

C. Rosier

### ABSTRACT

We consider a semilinear wave equation on a bounded interval. We propose a numerical scheme using a pseudo-spectral method for the spatial discretization and which gives both the control and the trajectory for the null controllability of a semilinear wave equation, without incorporating any damping or filtering. Our numerical scheme allows to test the validity of the classical fixed-point approach to design the control input when increasing the value of some parameter in the nonlinear part of the equation.

### TITLE

Computational approaches for periodic solutions of time delay systems and their stability: an experimental comparison.

### AUTHORS

Alessia Andò (1)

## Dimitri Breda (speaker, corresponding) (1)

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### **ABSTRACT**

Computing periodic solutions and assessing their stability is a key step in the analysis and control of dynamical systems. The task is particularly challenging in the case of systems involving delays, due to their intrinsic infinite dimension. Numerically, the standard approach to approximating periodic solutions of Delay Differential Equations (DDEs) relies on adaptive piecewise orthogonal collocation [5] implemented in a continuation framework as in DDE-BIFTOOL [6,7], in which the relevant Floquet multipliers are obtained as a by-product of the collocation. Alternatively, following [2] one can first reduce the original DDE to an ODE via pseudospectral collocation and then process the latter with standard continuation packages such as, e.g., MatCont [4]. More recently, pseudospectral collocation methods have been employed for the efficient continuation of delay equations [1] and also for discretizing monodromy operators to compute the multipliers by using the same adapted mesh on which the periodic solution has been computed [3]. In this work we select a number of test problems and perform an experimental comparison of these three methodologies, in view of understanding the peculiarities of each approach also in terms of their applicability with respect to the classes of equations, illustrating the reciprocal pros and cons and highlighting the computational challenges encountered especially when a strong mesh adaptation is required to compute the periodic solution.

### **REFERENCES**

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## TITLE

The role of time delay on the Kalman Conjecture

## AUTHORS

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## ABSTRACT

This presentation will discuss an open question: what class of systems with time delay satisfy the Kalman conjecture? In particular, there is no known example of a system that satisfies the Kalman conjecture without delay but does not satisfy the Kalman conjecture when delay is present. Nevertheless, the behaviour of even third order systems with delay is unknown. As time-delay system and saturation are very important from a practical point of view, understanding if their combination represents a higher deterioration on the stability of the system than their effects in isolation is a very interesting question.

The simpler example where this question is open is a second or third order system plus time delay. We believe that second order systems with time delay satisfy the Kalman conjecture. Recently, we have shown the construction of Zames-Falb multipliers for several cases [1], but a formal proof is still to be developed. This abstract summarises recent developments towards a formal proof of the above statement. The constructed multiplier is irrational, and the proof cannot be developed as in the non-delay case as it uses the pole location of the closed-loop system at the Nyquist gain. Hence a proof would be significantly different from the current proofs provided of the Kalman conjecture for different cases [2]-[4].

On the other hand, it is possible to develop frequency domain conditions to discard the existence a suitable Zames-Falb multiplier for a given linear system (see [5], [6] and references therein). By applying these conditions to third-order systems with delay, it is possible to find an example where the combination of saturation and delay produces unstable behaviour. These results demonstrate the suitability of the multiplier approach to understanding the stability issues when time-delay and saturation are present in the system.

## REFERENCES

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## TITLE

Predictor-Based Stabilization of a Large-Scale Robotic Network with Multiple Fixed Delays

## AUTHORS

Haonan Fan (1)

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## ABSTRACT

An input-output linearization combined with a predictor can help stabilize a network of nonlinear robots dynamics in the presence of delays. A key part of the stability proof of this distributed nonlinear system however requires to guarantee exponential stability of an arising linear-time invariant time delay system. For large scale networks, analysis of linear stability can be restrictive with the current state of the art. Our preliminary work indicates that we can perform this analysis in a computationally efficient manner and in parallel computation. We summarize the approach and present several case studies in this talk.

## TITLE

Finite-dimensional observer-based control of parabolic PDEs

## AUTHOR

Rami Katz and

**Emilia Fridman** (Tel Aviv University)

## ABSTRACT

Finite-dimensional observer-based controller design for PDEs is a challenging problem. Recently, a constructive LMI-based method for design of such controllers via modal decomposition approach was introduced for the 1D heat equation [1]. In this talk, such controllers will be presented for the linear 1D heat and Kuramoto-Sivashinsky equations, under both unbounded control and observation operators. We employ a modal decomposition approach via dynamic extension, using eigenvalues and eigenfunctions of a Sturm-Liouville operator. The controller dimension is dened by the number of unstable modes  $N_0$ , whereas the observer dimension  $N$  may be larger than  $N_0$ . We suggest a direct Lyapunov approach to the closed-loop system, which results in an LMI whose elements and dimension depend on  $N$ . The value of  $N$  and the decay rate are obtained from the LMI. We prove that the LMIs are

always feasible for large enough  $N$ . Robustness of finite-dimensional controllers with respect to input and output delays will be discussed.

## REFERENCES

[1] R. Katz and E. Fridman. Constructive method for finite-dimensional observer-based control of 1-D parabolic PDEs. *Automatica*, 122:109285, 2020.

## Wednesday Morning

### TITLE

Tracking control for  $2 \times 2$  linear hyperbolic PDEs that are bidirectionally coupled with nonlinear ODEs

### AUTHORS

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### ABSTRACT

A solution-based approach to trajectory tracking control is presented for  $2 \times 2$  linear heterodirectional hyperbolic PDEs that are actuated at one boundary and bidirectionally coupled with nonlinear ODEs at the other one. As discussed in [2], the control strategy relies on the solution of the PDE to essentially reduce the PDE-ODE system to nonlinear ODEs with delayed input. It is shown that a preliminary backstepping transformation simplifies this step significantly. Assuming differential flatness of the nonlinear boundary ODE system, a tracking controller exists for the reduced system for the case without an input delay. The flatness property allows to choose arbitrary reference trajectories for a flat output and to specify desired dynamics for the corresponding tracking error. To control the delay system based on the flatness-based feedback, a prediction of the ODE state is required, which follows from the solution of a Volterra integral equation. In the special case of linear ODEs and for an appropriate choice of design parameters, the presented solution-based controller for the PDE-ODE system is shown to be equivalent to the backstepping-based state feedback in [1].

### REFERENCES

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[2] A. Irscheid, N. Gehring, and J. Rudolph. Trajectory tracking control for a class of  $2 \times 2$  hyperbolic PDE-ODE systems. In 24th International Symposium on Mathematical Theory of Networks and Systems (MTNS), 2021. accepted.

#### TITLE

SDP relaxations of integral inequalities for stability and input-output analysis of PDE systems

#### AUTHOR:

Giovanni Fantuzzi (Department of Aeronautics, Imperial College London)

#### ABSTRACT

Lyapunov methods for the stability or input-output analysis of systems governed by partial differential equations (PDEs) often lead to intractable optimization problems where one or more integral functionals are constrained to be nonnegative on a function space. Valmorbida et al. ([IEEE Trans. Automat. Contr., 61:1649–1654, 2016](#)) proposed to use the divergence theorem in order to replace an integral inequality with a stronger pointwise one, which can be checked computationally by solving a semidefinite program (SDP) when the original integral inequality has a polynomial integrand. In a separate work, Korda et al. ([arXiv:1804.07565](#)) used occupation measures to relax the minimization of an integral functional into a convex problem, which can be further relaxed into an SDP in the case of a polynomial integrand. This talk will demonstrate that the verification of integral inequalities is closely related to the minimization of integral functionals, and that the seemingly different SDP relaxations developed in previous works are, in fact, weakly Lagrangian dual. Moreover, strong duality holds under certain compactness conditions. I will also outline new positive and negative results regarding the sharpness of these SDP relaxations, meaning their ability to yield arbitrarily accurate bounds on the minimum of an integral functional or the optimal value of an optimization problem with integral inequalities. The implications of these results for the stability and input-output analysis of PDE systems will be discussed and illustrated by means of examples. Part of this work is joint with David Goluskin (University of Victoria), Alexandr Chernyavskiy (University of Victoria) and Jason Bramburger (University of Washington).

#### REFERENCES

- [1] G. Valmorbida, M. Ahmadi, and A. Papachristodoulou, Stability analysis for a class of partial differential equations via semidefinite programming. *IEEE Trans. Automat. Contr.*, 61:1649–1654, 2016. DOI: [10.1109/TAC.2015.2479135](#)
- [2] M. Korda, D. Henrion, and J. B. Lasserre, Moments and convex optimization for analysis and control of nonlinear partial differential equations. [arXiv:1804.07565 \[math.OC\]](#), 2018.

#### TITLE

Further results on observer-based PI regulation control of a reaction-diffusion equation

#### AUTHORS:

Hugo Lhachemi

## Christophe Prieur

### **ABSTRACT:**

This chapter considers the output regulation control problem of a reaction-diffusion equation. Only boundary controls are used to solve this problem, and PI control strategy is developed. The suggested paradigm is based on a specific separation principle where a finite-dimensional controller and a finite-dimensional observer are designed separately and define the closed-loop system. The class of models include linear dynamics with Dirichlet and Neumann traces defining the outputs and the inputs, and also semilinear terms.

### **TITLE**

Forwarding approach for the stabilization of coupled PDE-ODE system

### **AUTHORS**

Swann Marx (LS2N)

Lucas Brivadis (LAGEPP)

Daniele Astolfi (LAGEPP)

### **ABSTRACT:**

This talk will be about a stabilization technique for a class of coupled PDE/ODE system. This technique relies mainly on the forwarding method, popularized in the 90's in the finite-dimensional nonlinear control community. Our technique is thus an extension of some finite-dimensional technique to the infinite-dimensional world. We will discuss on the structural properties needed to apply such a technique and on the introduction of mathematical objects, such as the infinite-dimensional Sylvester equation, which are essential for our analysis. Our technique will be illustrated on some systems of transport equations coupled with ODEs.

### **TITLE**

Stability analysis of the reaction-diffusion equation interconnected with a finite-dimensional system taking support on Legendre orthogonal basis

### **AUTHORS**

M. Bajodek

A. Seuret

F. Gouaisbaut

### **ABSTRACT**

The stability analysis of the reaction-diffusion subject to dynamic boundary conditions is not straightforward since the characteristic roots are unknown analytically. The goal of this chapter is to propose a numerical tool to evaluate the stability of the equilibrium of this class of infinite-dimensional systems. Following the general idea of finding numerically efficient criteria [2–4], a Lyapunov analysis is pursued. This chapter proposes an extension of [1], where Fourier-Legendre series allow to define an augmented finite-dimensional system, including the

dynamics of the first Fourier-Legendre coefficients, as well as the residual part. This new formalism induces the design of a Lyapunov functional split into a finite and an infinite-dimensional parts. From Bessel and Wirtinger inequalities applied to the Fourier-Legendre remainder and using its orthogonality properties, a simple condition of stability is obtained in terms of linear matrix inequalities. The stability condition is finally performed on examples and a discussion on the pros and cons of the method is provided. Keywords: Stability analysis · Infinite-dimensional systems · LMIs References

#### REFERENCES:

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#### TITLE

Sampled-data control for a class of linear hyperbolic systems

#### AUTHORS

Xinyong Wang,

Christophe Fiter,

Ying Tang,

Laurentiu Hetel

#### ABSTRACT

In this work, we discuss the stability for a class of linear hyperbolic systems [1] with distributed sampled-data controllers [2]. The original sampled-data system is firstly transformed into a new equivalent system by modelling the sampling induced error as a reset integrator operator (see [3] and references therein). Then by means of an appropriate Lyapunov function coupled with the Razumikhin technique [4], sufficient conditions are given for the  $R^\infty$ -stability [5] of the system based on linear matrix inequalities (LMIs). A numerical example illustrates our results: when the sampling interval is within the allowable range, the solution of the system converges from the domain of attraction to a positive invariant set.

#### REFERENCES

- [1] G. Bastin and J. M. Coron, *Stability and boundary stabilization of 1-D hyperbolic systems*. Springer, vol. 88, 2016.
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## Wednesday Afternoon

### TITLE

From propagation systems to time delays and back. Critical cases.

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### ABSTRACT

The paper originates from the early ideas of A. D. Myshkis and his co-workers [1, 2] and of K. L. Cooke and his co-worker [3, 4]. These ideas send to a one-to-one correspondence between lossless and/or distortionless propagation described by nonstandard boundary value problems and a system of coupled differential and difference equations with deviated argument. In this way any property obtained for one mathematical object is automatically projected back on the other one [5].

This approach is considered here for certain engineering applications [5, 6]. The common feature of these applications is the critical stability of the difference operator associated with the system with deviated argument obtained for each of the aforementioned applications. In fact the associated systems are of neutral type and, according to the assumption of Hale [7], only strong stability of the difference operator ensures robust asymptotic stability with respect to the delays.

If the difference operator is in the critical case, the stability becomes fragile with respect to the delays. Based on some old results in the field, a conjecture concerning the (quasi)-critical modes of the system is stated; also a connection with the so called *dissipative boundary conditions* [8] is suggested.

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## TITLE

Computational Modeling for 1D Distributed Parameter Systems

## AUTHOR

Daniela Danciu

ABSTRACT: The wide class of Distributed Parameter Systems (DPS) modeled by 1D parabolic and hyperbolic partial differential equations (PDEs) with non-standard boundary conditions (BCs) is mainly compound of systems arising from physics and control engineering applications. The common feature of all the systems within this class is that they display the so-called non-standard BCs. More precisely, this means that the system of PDEs is controlled by some BCs modeled by (possibly) nonlinear systems of ordinary differential equations (ODEs) which in turn are controlled by the system of PDEs. Due to the complexity of their mathematical models, nowadays a feasible and practical alternative to the analytical solution and system behavior evaluation are the numerical and computational solutions [1].

Consequently, the accuracy of the computational procedures used for deriving the approximate solution is of noticeable importance.

We shall present a systematic computational procedure previously introduced [2, 3, 4] to solve the class of systems referred above. This procedure, based on a convergent Method of Lines, ensures the convergence of the approximate numerical solution and also the preservation of the basic properties of the “true” solution as well as its Lyapunov stability. Thus, the approximate computational model allows numerical quantitative and qualitative analysis relevant to a specific problem. The computational efficiency of this procedure is ensured by its implementation by means of possibly massively, parallel structured devices

belonging to the Artificial Intelligence field – the cell-based recurrent neural networks. Some illustrative examples from control engineering applications include overhead crane with flexible rod [2], thermal power plants for co-generation [4], torsional vibrations of a drilling plant [6], hydroelectric power plants under waterhammer [8].

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#### TITLE

A systematic backstepping design of tracking controllers for ODE-PDE-ODE systems with nonlinear actuator dynamics

#### AUTHOR

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#### ABSTRACT

A backstepping design of tracking controllers is addressed for hyperbolic and parabolic bidirectionally coupled ODE-PDE-ODE systems with a nonlinear ODE at the actuated boundary. The stabilization of a reference trajectory is achieved by successively stabilizing the different ODE and PDE subsystems by means of virtual feedback laws. Using appropriate state transformations, the overall system is mapped into error coordinates from which asymptotic stability is easily inferred. As such, the approach suggested in [4] aims to take backstepping for PDEs back to its ODE origins (see e.g. [1]). In the end, the presented design solely relies on the strict feedback form of the ODE-PDE-ODE system. Using classical concepts such as integrator backstepping, output zeroing and differential flatness, well known in the ODE world, a multi-step design algorithm is shown to be essentially identical for the tracking control of both hyperbolic and parabolic ODE-PDE-ODE systems with spatially dependent coefficients and a nonlinear actuator ODE. This extends results in, e.g., [2, 3] and creates a multitude of possibilities for further generalizations.



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## **TITLE**

Indirect controller design for non-parabolic partial differential equations

## **AUTHORS**

**Kirsten Morris**

Orest Iftime

Hans Zwart

## **ABSTRACT**

There is by a well-known theory of approximation of PDEs where the semigroup is analytic. However, this assumption is not always satisfied. Of particular interest are a number of applications involving wave propagation. There are many different approximation methods for these systems that are fine for simulation. But the qualitative behaviour of the approximated eigenvalues can be quite different. Controller and estimator design with different approximations can yield different results. Of particular interest is optimal linear quadratic control of systems that may be only asymptotically stabilizable. For linear systems, this issue only arises with infinite-dimensional systems. Sufficient conditions which guarantee when approximations to the optimal feedback result in the cost converging to the optimal cost have recently been obtained.

## **TITLE**

Ramp metering: modeling, simulations and control issues

## **AUTHORS**

Cédric Join

Hassane Abouaïssa

Michel Fliess

## **ABSTRACT**

The aim of ramp metering is to improve the highway traffic conditions by an appropriate regulation of the inflow from the on-ramps to the highway mainstream. Our presentation

rests on several improvements: 1) Our simulation techniques do not need contrarily to other approaches any heuristic fundamental law. 2) There is no need of crucial time-varying quantities, like the critical density, which is most difficult to estimate correctly online. 3) Our feedback loop, which is stemming from model-free control, is easy to implement and shows an excellent robustness with respect to model mismatch. Several computer experiments are displayed and discussed.

## Thursday Morning

### TITLE

Critical delay as a measure of the difficulty of human balancing tasks

### AUTHORS

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### ABSTRACT

The relation between balancing performance on rolling balance board and reaction time is investigated. Ten young healthy adults performed balancing trials on a rolling balance board with different wheel radii and stance widths in the frontal plane. A 2- and a 3-degree-of-freedom rigid body models subject to delayed state feedback with a single lumped reaction delay were created to describe ankle and ankle-lumbar strategies. The critical time delay of the mechanical model is linked to the reaction time of the subjects in the following way. It is assumed that if the reaction time of a human subject is smaller than the critical delay corresponding to a fixed balance board geometry and stance width, then the central nervous system is able to find stabilizing control gains and the subject is able to stand on the balance board without falling. However, if the reaction time delay is larger than the critical one, then stabilization is not possible. The critical delays were determined by utilizing the multiplicity-induced dominance of the characteristic roots. Subjects' reaction time can therefore be estimated by evaluating the successfulness of balancing trials associated with different wheel radius and stance width parameters. The estimated reaction delays correlate with measured response times. The critical delay was found to be a good measure of the difficulty of the balancing task: the smaller the critical delay the more difficult the balancing task.

### TITLE

On Regional Sampled-Data Control of Nonlinear Time-Delay Systems with Input Saturation Constraints

### AUTHORS

M. Di Ferdinando

P. Pepe

et al.

### **ABSTRACT**

In this chapter, we deal with the problem of the stabilization in the sample-and-hold sense by emulation of continuous-time dynamic output feedback controllers with input saturation constraints. Nonlinear time-delay systems not necessarily affine in the control input are studied. Sufficient conditions are provided such that the emulation of continuous-time dynamic output feedback controllers with amplitude bounds yields stabilization in the sample-and-hold sense limited to suitably small regions. The inter-sampling system behavior as well as time-varying sampling intervals are taken into account. The case of nonlinear delay-free systems is addressed as a special case. In this case, the sufficient conditions, ensuring the local stabilization in the sample-and-hold sense, are satisfied if the continuous-time dynamic output feedback controller is a global stabilizer. An example is presented which validates the theoretical results.

### **TITLE**

Recent results on the analysis and estimation of linear switched models by their equivalence to delay systems

### **AUTHORS**

**Vittorio De Luliis (corresponding author)**

Alessandro D'Innocenzo

Costanzo Manes

### **ABSTRACT**

This contribution surveys some recent results [1], [2] on a class of linear switched models leveraging their equivalence to linear discrete-time delay systems. Such relationship, which has been extensively used in the literature on networked control systems [3], allows to address the stability analysis of delay systems using tools from switched systems theory, and viceversa. Whereas most of literature has focused on the delay-dependent analysis (see e.g. [4]), our work exploits the peculiar properties of linear positive delay systems, for which the stability is not affected by the magnitude of the delays. We illustrate how this property reflects into equivalent results on some classes of discrete-time switched systems whose dynamic matrices are in block companion form. First, we address the case of positive switched systems, for which we propose a comparison of our results with the existing literature on linear common Lyapunov functions. Then we remove the positivity assumption, yet at the expense of introducing conservatism. We further discuss how, under some special combinatorial assumptions on the switching matrices, the arbitrary switching stability reduces to a condition involving a single matrix.

Some consequences on the problem of deciding whether the joint spectral radius of matrices in block companion form is less than one are illustrated. Finally, we present an application to switched autoregressive models with exogenous inputs (switched ARX models), a very popular choice in system identification, which leads to simple constraints for the problem of learning a dynamical switched model with stability guarantees [5].

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## TITLE

Stability, robustness analysis and model order reduction of periodic control systems with delay

## AUTHORS

Wim Michiels

Linear periodic time-delay (LPTD) models describe dynamical systems in all fields of science and engineering. They arise from the linearization of nonlinear autonomous time-delay systems around a periodic orbit or from a periodic excitation. At the same time, in some situations periodic control strategies outperform time-invariant controllers and even solve tasks beyond their scope, such as stabilization in fixed time by smooth feedback. The control design for LPTD systems is challenging, due to the interplay between the infinite-dimensional dynamics and the explicit time-dependence of the models, which renders powerful techniques for spectrum based control of time-invariant delay systems not directly applicable, as the latter techniques are rooted in the availability of a characteristic equation in explicit form. This talk gives an overview of basis properties, limitations and opportunities of control of LPTD systems and of some first steps towards the development of systematic analysis, model reduction and control design tools.

## TITLE

Prediction-based controller for control systems with stochastic input delay

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## ABSTRACT

This talk focuses on dynamical systems with a stochastic input delay, which are ubiquitous in engineering systems, due to uncertainties in the transmission path. For input-delay systems, the use of prediction-based control methods is quite standard. This method was first used for constant delay in [6] and [1] before being extended to time-varying delays (see [2,7]). Yet, the extension of prediction design to handle stochastic delays is still an open problem.

On the other hand, [4] studies the stability of linear systems with stochastic time delay, and proposes the use of Markov process to describe the delay. [3] further extends the research to nonlinear systems. However, the authors only considered state delay. We ground our approach for prediction-based control on this modeling. In this talk, following [3,4], we propose to describe the stochastic input delay as a continuous-time Markov process with a finite number of values. The stochastic input delay of the system is compensated through the controller based on constant-horizon prediction. By applying the method of backstepping transformation in [5], we obtain that exponential stabilization of the closed-loop system is achieved, under a sufficient condition. This condition bears on the range of the possible values of the time lag, which should be sufficiently small and close enough to the prediction horizon. We will start with linear systems and gradually extend the conclusions to nonlinear systems.

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## TITLE

Consensus and control of delayed opinion formation models

## AUTHORS

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## ABSTRACT

The Hegselmann-Krause model has been introduced to describe the dynamics of opinions in a population of  $N$  agents. It is natural to introduce a time delay in the model to take into account a reaction time or a time for each agent to receive information from other agents. We will consider a Hegselmann-Krause model with non-symmetric interaction potential and time-dependent time delay. By using a Lyapunov functional approach, we will prove convergence to consensus if the time delay satisfies a suitable smallness assumption. The infinite-dimensional model, obtained by taking the mean field limit of the discrete system, is also analyzed. Using the fact that the constants appearing in the consensus estimates for the particle model are independent of the number of agents  $N$ , we can find an exponential consensus result for the continuum model. A control problem for a delayed Hegselmann-Krause model with leadership is also discussed. Finally, some numerical tests are illustrated.

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## Thursday Afternoon

### TITLE

Fractional functional differential equations and generalized ODEs

### AUTHORS

Paulo Carvalho

Carolina Lafetá

Carlos Lizama

Jaqueline Godoy Mesquita (speaker, corresponding)

### ABSTRACT

In this work, we will show a correspondence between the solutions of fractional functional differential equations and the solutions of generalized ODEs, and some applications from this correspondence.

### TITLE

Energy shaping control of 1D Distributed Parameter Systems

### AUTHORS

Y. Le Gorrec

H. Ramirez;  
Y. Wu;  
N. Liu;  
A. Macchelli;

**ABSTRACT:** In this chapter we give an overview on energy shaping control for Distributed Parameter Systems defined on a 1D spatial domain using the port Hamiltonian framework. We consider two different cases: when actuators and sensors are located within the spatial domain and when the actuator is situated at the boundary of the spatial domain, leading to a boundary control system (BCS). In the first case we show how dynamic extensions and structural invariants can be used to change the internal properties of the system when the system is fully actuated, and how it can be done in an approximate way when the system is actuated using piecewise continuous actuators stemming from the use of patches. Asymptotic stability is achieved using damping injection. In the boundary controlled case we show how the closed loop energy function can be partially shaped, modifying the minimum and a part of the shape of this function and how damping injection can be used to guarantee asymptotic convergence.

#### TITLE

Output regulation and tracking for Linear ODE-Hyperbolic PDE-ODE systems

#### AUTHORS:

Jeanne Redaud

Federico Bribiesca Argomedo

Jean Auriol

#### ABSTRACT

In this talk, we consider the problem of output regulation and output tracking for a linear  $2 \times 2$  hyperbolic PDE with actuation and load dynamics. The main focus is on load tracking (i.e. tracking an output depending on the states of the unactuated ODE, while only controlling the opposite ODE system). The problem of output stabilization has been studied for instance in [4] and [5], and an output regulation problem for related systems (without actuator and load dynamics) is considered in [1]. Here, we propose to extend these results to a class of systems similar to that considered in [3] and extending the filtering techniques to a dynamically augmented system with finite-dimensional exo-systems considering of possible trajectory and disturbance inputs. As in the aforementioned reference, issues with respect to small delays in the state reconstruction and feedback loop are considered. Due to the nature of the disturbances, the problem of state and disturbance reconstruction from a different output (i.e. anti-collocated measurement and tracking) is also considered.

The considered scenario finds applications in many systems of engineering interest, such as drilling systems [2], pneumatic systems [6], and electric transmission lines [7].

#### REFERENCES:

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## TITLE

Nonlinear time-delay algorithms for payload oscillation damping – analysis and applications

## AUTHORS

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## ABSTRACT

The contribution aims at analysis and implementation aspects of nonlinear time-delay controllers for damping oscillations of suspended payload under the motion constraints of the pivot yielding the uncontrollability from the linear control system point of view. In particular, the configuration with a fixed position pivot is considered as the first option. In this case, the payload damping is performed via adjusting the suspension length. Starting from the pioneering work by Stilling and Szyszkowski [1], nonlinear time delay controllers were proposed by the authors' team in [2] to generate the harmonic trajectory with a double frequency compared to the frequency of the payload oscillations, see also [3] presenting a comparison with nondelayed algorithms. The second considered configuration stems in considering the suspension length fixed and the oscillation damping performed by up and down motion of the pivot. Starting from the theory proposed in [2], the nonlinear time delay controllers were



adjusted for this modified configuration in [4]. In this presentation, both the configurations are compared and their application potential is discussed in a broader context. Besides, the force, energy needs and stability aspects are investigated in more detail. In addition to the original design concepts proposed in [2] and [4], including an additional distributed delay to the nonlinear control scheme, the oscillation amplitude is adjusted at the final stage of the control action cycle. By this adjustment, compared to [2] and [4] where the motion was stepwise stopped by an embedded logic, the algorithm has been fully automated. Both simulation and experimental validation of the adjusted control schemes are included.

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## Friday Morning

### TITLE

Event-triggered output feedback control for simultaneous stabilization of traffic flow on connected roads

### AUTHORS

N. Espitia

J. Auriol

H., Yu

M. Krstic

### ABSTRACT

In this talk, we develop an event-triggered boundary output feedback control for simultaneous stabilization of traffic flow on connected roads.

The traffic dynamics of density and velocity are described with the linearized Aw-Rascle-Zhang (ARZ) macroscopic traffic partial differential equation (PDE) model which results in a coupled hyperbolic system. The control objective is to simultaneously stabilize the upstream and downstream traffic to a given spatially-uniform constant steady-state that is in the congested

regime. To suppress stop-and-go traffic oscillations on the cascaded roads, we consider a ramp metering strategy that regulates the traffic flow rate entering from the on-ramp to the mainline freeway. The ramp metering is located at the outlet with only boundary measurements of flow rate and velocity.

The main idea is that the control signal is only updated when an event triggering condition is satisfied. Compared with the continuous input signal, the event-triggered boundary output control presents a more realistic setting to implement by ramp metering on a digital platform. The event-triggered boundary output control design relies on the emulation of the backstepping boundary output feedback and on a dynamic event-triggered strategy to determine the time instants at which the control value must be updated. We prove that under this strategy there exists a uniform minimal dwell-time (independent of initial conditions), thus avoiding the so-called Zeno phenomenon and we guarantee the exponential convergence of the closed-loop system under the proposed event-triggered boundary control. A numerical example illustrates the results.

## TITLE

PDE and DDE models for dynamic analysis of vessel-valve-pipe system

## AUTHORS

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## ABSTRACT

Pressure relief valves are widely used to protect high-pressure systems with various feasible setups regarding the environment where the valve is installed. The current system in question consists of a vessel, a direct spring-operated pressure relief valve and a downstream pipe. The vessel-valve subsystem can be described with the help of three first order ordinary differential equations [1]. The pipe is modelled by means of the Navier-Stokes equation and the continuity equation by neglecting the friction and the convective terms. There are two boundary conditions at the inlet and outlet of the pipe. This distributed parameter system adds two more equations to the vessel-valve subsystem. This coupled system of ordinary (ODEs) and partial differential equations (PDEs) can be transformed to delay differential equations (DDEs) with the help of a travelling wave solution [2]. The time delay corresponds to the time needed for the pressure wave running along the pipe to and from.

Linear stability analysis is carried out including the calculation of the frequencies and mode shapes of the emerging self-excited vibrations. Stability charts are presented in the parameter plane of the pipe length and vessel volume. The vibration frequencies and the stability boundaries are expressed in closed form. These offer opportunities for qualitative and quantitative validations of the model. Comparison to numerical simulations of the PDEs confirm the analytical results.

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## TITLE

Stability and Bifurcation Analysis for an Extended Model of Gene Regulatory Network with Delays

## AUTHORS

Dilan Öztürk

Fatihcan Atay

Hitay Özbay

## ABSTRACT

Using nonlinear delay differential equations in the analysis of gene expression data provides us useful insights into the genetic regulatory mechanisms in living beings. The ongoing interest for a cyclic biological gene regulatory network (GRN) model is to determine the stability and bifurcation criteria with time delay. We aim to investigate an extension of a cyclic gene regulatory model which has been studied earlier and present the analysis of this nonlinear time-delayed GRN model with second order subsystems considering Hurwitz matrices in the system analysis. The delay dependent stability condition for the linearized extended system using the characteristic equation is provided. We also examine the Hopf bifurcation of the extended structure using different delay structures. Bifurcation analysis is made using both only discrete time delays and mixed time delays including discrete and Gamma-type distributed delays. For the extended cyclic GRN model containing a nonlinear feedback loop and time delays, simulations are performed with different parameters to deepen our analysis.

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**TITLE**

Controlled invariance of systems with time delays

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**ABSTRACT**

We reconsider the question of controlled invariance for a class of continuous systems with time delays. Many control problems come down to the identification of regions that are invariant for the closed-loop system, and designing an ad hoc control law is the key of the design procedure. If the system is subject to delays, the solution depends on initial conditions, that are functions over a finite horizon. The initial conditions are associated to the state or to the inputs, depending on the delays, and they have to be taken into account in the design of controlled invariant regions. We propose a novel definition of controlled invariance for systems with delays, that is based on the existence of input initial conditions, in addition to an input function, that lets the state trajectory remaining in the region that is considered. We show that this concept of controlled invariance is equivalent to the existence of convenient control law and initial conditions, that permit to force the invariance. We illustrate the method in practical situation, arising from manufacturing and production management. The method can be interpreted in terms of prediction, and we discuss the causal implementation of the corresponding feedback.

**TITLE**

The optimal time for the controllability of hyperbolic systems in one dimensional space

**AUTHOR**

Hoai-Minh Nguyen

**ABSTRACT**

We discuss the optimal time for the controllability of hyperbolic systems in one dimensional space of time-invariant speeds with one-side controls. Both time-independent and time-varying zero-order coefficients are considered. Even the zero-order coefficients have no effect on the characteristic flows of the control system, surprisingly, the optimal times are significantly different in the time-independent and time-varying settings. However, when the analyticity with respect to time is imposed for the zero-order coefficients in the time-varying setting, the optimal time in this setting is then shown to be the same as the one in the time-independent setting. The later result can be then viewed as an extension of a well-known controllability property of linear differential equations: if a linear control system is controllable

in some positive time and is analytic, then it is controllable in any time greater than the optimal time, which is 0. This is based on joint work with Jean-Michel Coron.

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## TITLE

Two recent stability analysis techniques

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## ABSTRACT

We present two recent stability analysis techniques for broad families of nonlinear dynamical systems. The first one is called ‘trajectory based approach’ and, in contrast with Lyapunov based approaches, it involves verifying certain inequalities along solutions of auxiliary systems. It is especially useful when systems with delays and discontinuities are studied. The second technique involves inequalities of Halanay’s type. It is especially efficient for time-varying dynamical systems. We show in particular how this technique can be extended to the case where vectorial inequalities are satisfied. Both of the techniques apply to a wide range of systems, notably time-varying systems with time-varying delay, ODE coupled with difference equations, and networked control systems with delay.